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Predator Effect of Atlantic Bonito on the Black Sea Anchovy and Their Sustainable and Optimal Fishery

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Abstract

The Atlantic bonito (*Sarda sarda*) and Black Sea anchovy (*Engraulis encrasicolus*) fisheries in the southern part of the Black Sea currently lack a consistent harvesting strategy. To address this issue, a fishery model has been developed to optimize and stabilize the predator-prey relationship between these two species. This study offers an optimal and stable predator-prey relationship, resulting in higher landings and profits for the commercial fishery compared to the current harvesting strategy outlined in the study. The findings of this research can be utilized to determine sustainable yields and fishing quotas for these fisheries. Additionally, the examination of the predator-prey relationship between these species has revealed that, on an annual average, 93,259 tonnes of anchovy are consumed by the Atlantic bonito, which corresponds to approximately 46% of the estimated average anchovy landing. Furthermore, it has been observed that a 10% increase or decrease in the Atlantic bonito's feeding habits on the anchovy population leads to fluctuations in anchovy landings of up to 13% and anchovy profits of up to 40%

Introduction

The Atlantic bonito and Black Sea anchovy are highly valued and important species for commercial fishing in the southern part of the Black Sea. These two species enter the southern part of the Black Sea from different regions at different times. The Atlantic bonito starts migrating into the Black Sea from the Aegean Sea and the Marmara Sea for spawning and feeding from the end of April to the middle of August and then returns to those regions in November and December. The commercial fishing season for Atlantic bonito in the southern part of the Black Sea typically runs from September to November, with the highest levels of fishing occurring in September and October. The Black Sea anchovy, on the other hand, mostly stays in the northern part of the Black Sea during the spawning period and only enters the southern part in October, with the highest levels of fishing occurring in November and December (Zengin and Dincer, 2006; Gucu et al., 2017). Therefore, the predator-prey interaction between these two species mainly occurs between October and December in the southern part of the Black Sea.

The lack of regulations regarding the maximum or sustainable amount of landings for the Atlantic bonito and Black Sea anchovy has resulted in significant fluctuations in the amount of landings taken each year in the Black Sea. For instance, the annual landing of Atlantic bonito ranges from 603 tonnes to 68,830 tonnes in years 2019 and 2005 respectively. This situation not only leads to ecological losses but also economic losses due to either high fishing efforts for catching a small amount of fish or overfishing beyond the natural reproduction capacity of the fish population for upcoming seasons. The same issue is also observed for the Black Sea anchovy. As a result, it is necessary to implement sustainable and ecologically friendly management strategies for the Black Sea fishery to ensure reproductive ecological systems and profitable fisheries. (Lauck et al., 1998; Foley, 2013; Demir and Lenhart, 2019). As an alternative, this study proposes building a two-species mathematical model and applying stability analysis to ensure that the predator-prey relationship of the species remains stable. This method mainly requires landing data to be implemented. Using mathematical models for fishery management is not a new approach, but it is insufficient to fully comprehend species dynamics and avoid overfishing. Therefore, implementing stability analysis is crucial to understanding species dynamics and avoiding overfishing (Panja and Mondal 2015; Bentounsi et al., 2017; Agmour et al., 2018; Demir and Lenhart, 2019; Demir and Lenhart, 2021).

The study aims to analyze the predator-prey relationship between the Atlantic bonito and the Black Sea anchovy through a mathematical model that employs stability analyses. The model is then coupled with optimal control theory to achieve both sustainable and optimal landing for these fisheries. The same approach is proposed by Demir and Lenhart, 2019. The methods section outlines the details of the fishery model, stability analyses, and optimal control theory, while the results section presents the findings based on stability conditions and optimal control conditions. The study concludes by discussing the benefits of harvesting at the maximum and optimal sustainable levels and how profitable these fisheries can be compared to the current harvesting strategy simulated in the study in section 3.1.

Effective management of commercial fisheries is a critical issue that scientists and experts must address to ensure sustainable fishery practices worldwide. Fish populations are currently facing overfishing due to incorrect harvesting strategies or high exploitation rates, emphasizing the need for ecosystem-friendly management strategies to achieve resilient and sustainable fish stocks (Hilborn, 2012; Bardey, 2019). Despite the significance of fish management, many current studies fail to consider the current status of fish populations or account for predator-prey relations and stability analysis, which are essential for sustainable fishery management. Ignoring these factors may result in overfishing or even the collapse of fish populations. Therefore, it is critical to consider predator-prey relations and stability analysis when developing management strategies for fisheries to promote sustainable practices (Lauck et al., 1998; Foley, 2013; Demir and Lenhart, 2019).

There are several stock assessment methods that can be used to determine the status of fish stocks, such as XSA, VPA, BMS, and CMSY. However, these methods require various types of data, such as diet data, natural mortality, fishing mortality, abundance index of species, predator ratio estimates, and more. Obtaining these data is often time-consuming and expensive. Additionally, these assessment methods utilize a single species modeling framework that disregards predator-prey relationships, resulting in overestimates for biomass and maximum sustainable yield for fish stocks.

Material and Methods

The study utilized a mathematical model (Eq.1) to understand the predator-prey relationship between Atlantic bonito and Black Sea anchovy. The first step was to ensure a sustainable and stable predator-prey relationship by investigating the system's stability. The study then coupled the fishery model with Optimal Control Theory (OCT) and used constant harvest rates with an upper bound based on the stability analysis of the predator-prey system. The current status of the two species was then simulated, and the optimal control strategy that maximized net profit was determined. Our analysis depends on annual landing data of these species between years 2000 and 2022 (STEFC, 2017; TUIK, 2023), annual average prices of these species (TUIK, 2023), and the annual average exchange rate from Turkish Liras to US Dollar (Turkish Central Bank, 2023). Details and additional information about model setup are provided in the following subsections.

Model Formulation and Description

The system's behavior, which includes the predator-prey relationship between the Black Sea anchovy (A) and the Atlantic bonito (B), is described using ordinary differential equations. This study also takes into account the harvesting of anchovy, with the harvesting term represented by $h_1(t)A(t)$ where h1(t) is the harvest rate and $A(t)$ represents the amount of anchovy in the system at time t. Similarly, the harvesting term for B is represented by $h_2(t)B$(t). Table 1 provides a description of the parameters used in the model.

$$\frac{dA}{dt} = rA\left(1-\frac{A}{K}\right) - aAB - h_1A$$

$$\frac{dB}{dt} = bAB - h_2B \qquad (1)$$

with initial conditions: $A(0) = A_0$ and $B(0) = B_0$. The terms aAB and bAB in the model represent the relationship between predator (Atlantic bonito) and prey (Black Sea anchovy) populations. The anchovy population growth rate is modeled using a logistic equation with an intrinsic growth rate of r and a carrying capacity of K. All the coefficients and initial conditions in the model are positive and have upper bounds. It should be noted that commercial fishing activities for these species take place mainly between September and January, with the interaction between the species occurring primarily between October and December in the southern region of the Black Sea. Therefore, the predator-prey relationship between these species is considered only for this 90-day period in this study.

The fishery model (1) was constructed with a logistic growth rate for the anchovy population since there are many predators and prey of anchovy, while all other interactions, except for the Atlantic bonito, were assumed to be hidden in this logistic growth rate. As the Atlantic bonito is one of the top predators in the southern part of the Black Sea and approximately %63 of bonito diets come from anchovy predation (Daskalov et al., 2020), a logistic growth rate was not considered for the Atlantic bonito population. Instead, the growth of bonito was assumed to come from other species, accounting for approximately %37 of bonito growth. After parameter estimation of the model, the coefficient of bAB was updated to be $(b + 37b/63)AB$. Note that since the Atlantic bonito is one of the top predators in the Black Sea, we ignore its predation by not including logistic growth. We also changed the value of %63 to be its 10 percent below (%56.7) and above (%69.3) to see the effect of these changes on the predator prey relation of species. We are not going to make a separate parameter estimation for these new cases and redo all the analysis, instead we change the coefficient of the term bAB by increasing or reducing the parameter b, 10 percent below or above its estimated value. Thus, we keep all the parameters the same except for b in these new analyses (see Table 3 for these cases).

Since the anchovy equation in (1) has a common factor of *A* on its right-hand side and has negative terms, we can obtain a uniform bound for the anchovy equation as *0 < A ≤ M1* for the positive and bounded initial condition of *A* since $\frac{dA}{dt} \le rA$. Similarly, we then can obtain a dt uniform bound for the bonito equation as *0 < B ≤ M2* since we can find a bound on *B* by using the boundedness of *bB*. Thus, model outputs will stay positive and bounded in this study.

Stability Analysis of the Model

We are now going to investigate stability of the model that is described by Equation (1). To do this, we begin by setting the time derivatives in Equation (1) equal to zero, which allows us to determine the equilibrium points of the system.

We then rescale Equation (1) and arrive at a simplified model, which is presented as follows. For details obtaining the Equation (2) from the Equation (1), see the appendix.

$$\frac{dx}{dt} = (1 - y - (1+\delta)x)$$

$$\frac{dy}{dt} = \beta y(x - a) \qquad (2)$$

where $A = xK$, $y = \frac{ab}{r}$, $\beta = \frac{bK}{r}$, $\alpha = \frac{h_2}{bK}$, $\delta = \frac{h_1}{r}$, and t is replaced with $\frac{t}{r}$. When we set $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$, then we will get the equilibrium points of the model in the order $E = (x^*, y^*)$ as $E_1 = (0,0)$, $E_2 = (\frac{1}{1+\delta}, 0)$, and $E_3 = (\alpha, 1 - (1+\delta)\alpha)$. The coexistence equilibrium point E_3 is positive and biologically feasible when $\alpha > 0$ and $(1+\delta)\alpha < 1$. To investigate the stability of these equilibrium points, let's first find the Jacobian (community) matrix. We will set $\frac{dx}{dt} = f_1(x,y)$ and $\frac{dy}{dt} = f_2(x,y)$ to obtain the jacobian matrix, *J* as below

$$J_{(x^*,y^*)} = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 1 - y^* - 2(1+\delta)x^* & -x^* \\ \beta y^* & \beta(x^* - \alpha) \end{pmatrix}$$

At the equilibrium point, $(x^*, y^*) = (0,0)$

$$J_{(0,0)} = \begin{pmatrix} 1 & -0 \\ 0 & -\beta\alpha \end{pmatrix}$$

This matrix has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -\beta\alpha$ ($0 < \beta$ *and* $0 < \alpha$). Note that when both eigenvalues are negative, then the equilibrium point is stable, otherwise unstable. Thus, the equilibrium point, $(x^*, y^*) = (0,0)$ is unstable. At the equilibrium point, $(x^*, y^*) = (\frac{1}{1+\delta}, 0)$

$$J_{(\frac{1}{1+\delta},0)} = \begin{pmatrix} -1 & -\frac{1}{1+\delta} \\ 0 & \beta(\frac{1}{1+\delta} - \alpha) \end{pmatrix}$$

The eigenvalues are $\lambda_1 = -1$ and $\lambda_2 = \beta(\frac{1}{1+\delta} - \alpha)$. When $\alpha > \frac{1}{1+\delta}$, then the equilibrium point is a stable node. If $\alpha < \frac{1}{1+\delta}$, then the equilibrium point is an unstable node. Lastly, at the equilibrium point, $(x^*, y^*) = (\alpha, 1 - \alpha(1+\delta))$, the Jacobian matrix will be in the form

$$J_{(\alpha,1-(1+\delta)\alpha)} = \begin{pmatrix} -(1+\delta)\alpha & -\alpha \\ \beta - \beta\alpha(1+\delta) & 0 \end{pmatrix}.$$

To obtain eigenvalues, we solve the determinate of the above matrix and obtain the following quadratic polynomial as

$$\lambda^2 + (1+\delta)\alpha\lambda + \lambda\,\beta(1-\alpha(1+\delta)) = 0$$

By the Routh-Hurwitz stability criterion, this coexistence equilibrium point is stable if $\alpha(1+\delta) < 1$ and unstable if $\alpha(1+\delta) > 1$. The eigenvalues are obtained by the discriminate rule for quadratic equation given above as

$$\lambda_{1,2} = \frac{-\alpha(1+\delta) \mp \sqrt{\alpha^2(1+\delta)^2 - 4\alpha\beta(1-\alpha(1+\delta))}}{2}$$

When $\alpha > \frac{4\beta}{(1+\delta)^2(1+\frac{4\beta}{1+\delta})}$, then we have a stable node. If $\alpha < \frac{4\beta}{(1+\delta)^2(1+\frac{4\beta}{1+\delta})}$, we have a stable focus. Note that $\alpha(1+\delta) < 1$ will be the constraint for a stable predator-prey relation between these species and the parameter estimation of the fishery model (1) will be conditional on this constraint as well. Depending on the stability constraints, we estimated the parameters given in the Table 1 with estimated harvest rates, $h_1 = 0.36$ and $h_2 = 0.22$.

Optimal Control for Harvesting Strategies

To account for seasonal harvesting in the southern Black Sea, we introduce seasonality in the control variables (harvest rates) by defining the union of time intervals as $\Omega = \bigcup_{i=1}^{n}[a_i, b_i]$, where n represents the number of years and the interval $[a_i, b_i]$ represents the fishery season in year $i = 1, 2, \ldots, n$. As a result, the fishery season occurs on the set $[0, T]$, and the controls h_1 and h_2 are set to zero on the set $[0, T]\backslash\Omega$. In the commercial fishery of the Black Sea anchovy and the Atlantic bonito, both fishing seasons last for about three months. For the Atlantic bonito, fishing starts on September 1 and mostly ends by the end of November, while for the anchovy fishery, it starts in November and ends by the end of January (Zengin and Dincer, 2006; Gucu et al., 2017). Thus, we consider harvesting the Atlantic bonito from September to November (90 days) and the anchovy from November to January (90 days). The objective functional for system (1) with its initial conditions are

$$\begin{aligned} J(h_1, h_2) &= \int_0^T e^{-\alpha t}(p_1 h_1 A + p_2 h_2 B - (\mu_1 + \mu_2 h_1)h_1 - (\mu_3 + \mu_4 h_2)h_2)dt \\ &= \int_\Omega e^{-\alpha t}(p_1 h_1 A + p_2 h_2 B - (\mu_1 + \mu_2 h_1)h_1 - (\mu_3 + \mu_4 h_2)h_2)dt \qquad (3) \end{aligned}$$

where $J(h_1, h_2)$ is the discounted net profit value of both fisheries, h1 and h2 are the harvest rates (the control variables), and $e^{-\alpha t}$ denotes the discount rate with interest rate α. The term $e^{-\alpha t}(p_1 h_1 A + p_2 h_2 B)$ represents the revenue of the fishery with the price p_1 for the anchovy and p_2 for the bonito, and the terms

$$e^{-\alpha t}\big((\mu_1 + \mu_2 h_1)h_1 + (\mu_3 + \mu_4 h_2)h_2\big)$$

represents the cost of the fisheries. This cost term is nonlinear in h_1 and h_2, but the coefficients μ_2 and μ_4 are taken small enough to have a small effect on numerical calculations. Note the costs in the objective functions have been frequently represented by nonlinear terms in the harvest control (Kelly et al., 2019; Neubert, 2008; Herrera et al., 2016). One possible reason for this quadratic term is the additional cost associated with the interference between vessels and their workers while fishing near the same location (Herrera et al., 2016). The coefficients $p_1\, and\, p_2$ are annual anchovy prices between years 2000 and 2022 provided from Turkish Statistical Institute (TUIK, 2023). Our purpose is to find optimal controls, h_1^* and h_2^* in $\mathcal{A}$ such that

Table 1. Parameter description and values in the case of constant harvesting strategies. Here e is a scientific notation in MATLAB and it is a shorthand for 10

Parameters	Descriptions	Value	Source
A_0	Initial biomass of anchovy, A	128,621	Estimated
B_0	Initial biomass of bonito, B	17,753	Estimated
r	Intrinsic growth rate of anchovy, A	0.53	Estimated
K	Carrying capacity of anchovy, A	973,772	Estimated
a	Consumption rate of A due to its predator B	$2.9e^{-5}$	Estimated
b	Growth rate of B due to predation of A	$3.8e^{-7}$	Estimated
h_1	Harvest rate of anchovy, A	vary	Estimated
h_2	Harvest rate of bonito, B	vary	Estimated
μ_1	Linear coefficient of anchovy cost	\$5,760,000	Assumed
μ_2	Quadratic coefficient of anchovy cost	\$0.01	Assumed
μ_3	Linear coefficient of bonito cost	\$2,695,000	Assumed
μ_4	Quadratic coefficient of bonito cost	\$0.01	Assumed
α	The interest rate of the discount rate	0.01	Assumed

(*) The constant harvest rates are estimated as $h_1 = 0.36$ and $h_2 = 0.22$

$$J(h_1^*, h_2^*) = \sup_{h_1,h_2 \in \mathcal{A}} J(h_1, h_2)$$

Where $\mathcal{A}$ is the class of admissible controls such that $\mathcal{A} = \{(h_1, h_2): [0,T] \rightarrow [0,M] \mid h_i = 0 \; on \; [0,T] \setminus \Omega, and \; h_i$ *and Lebesque measurable for* $i = 1,2\}$.

We conducted a numerical stability analysis for the fishery model (1) by varying the values of the harvest rates h_1 and h_2 using the parameter values obtained from the parameter estimation section. The coexistence equilibrium point $E = (A^*, B^*)$ is biologically feasible and stable when the constant harvest rates h_1 and h_2 are both less than or equal to 0.36 and 0.22, respectively. Hence, we consider these values as critical upper bounds for ensuring sustainable fisheries of the Atlantic bonito and the Black Sea anchovy. The necessary conditions satisfied by optimal controls and their corresponding states will be driven by using Pontryagin's Maximum Principle (Pontryagin, 1967).

Theorem 2.1 (Existence of an Optimal Control).

There exist optimal controls h_1^* and h_2^* in the class of admissible controls A, which maximizes the objective functional $J(h_1, h_2)$ subject to the state system (1) with its initial conditions.

Theorem 2.2.

Given optimal controls h_1^* and h_2^*, and the state solutions A^* and $B^*$of the system (1), there exist adjoint variables λ_A and λ_B corresponding to A^* and B^* respectively, which satisfy the following equations:

$$\frac{d\lambda_A}{dt} = -\left(e^{-\alpha t} p_1 h_1 + \lambda_A \left[r - \frac{2rA^*}{K} - \alpha B^* - h_1\right] + b\lambda_B B^*\right)$$

$$\frac{d\lambda_B}{dt} = -(e^{-\alpha t} p_2 h_2 + \alpha A^* \lambda_A + (bA^* - h_2)\lambda_B) \quad (4)$$

with the transversality conditions: $\lambda_A(T) = 0$ and $\lambda_B(T) = 0$.(5)

Furthermore, characterizations of optimal controls h_1^* *and* h_2^* are given by

$$h_1^*(t) = min\left\{M, max\left\{0, \frac{A^*(p_1 - e^{-\alpha t}\lambda_A^*) - \mu_1}{2\mu_2}\right\}\right\} \text{ on } [0,T]$$

$$h_2^*(t) = min\left\{M, max\left\{0, \frac{B^*(p_1 - e^{-\alpha t}\lambda_B^*) - \mu_3}{2\mu_4}\right\}\right\} \text{ on } [0,T] \quad (6)$$

The adjoint equations (4) and the characterization of optimal controls (6) given in Theorem 2.2 are obtained from the Hamiltonian:

$$H = e^{-\alpha t}(p_1 h_1 A + p_2 h_2 B - (\mu_1 + \mu_2 h_1)h_1 - (\mu_3 + \mu_4 h_2)h_2) + \lambda_A\left[rA - \frac{rA^2}{K} - \alpha AB - h_1 A\right] + \lambda_B[bAB - h_2 B]. (7)$$

The adjoint equations were optioned from the partial derivatives of Hamiltonian with respect to state variables and the characterization of optimal controls was obtained from the partial derivatives of Hamiltonian with respect to control variables. For the details in proofs of Theorem 2.1 and 2.2, see the study proposed by Demir and Lenhart, 2019 since similar proofs for Theorem 2.1 and Theorem 2.2 are presented in this study.

The optimality system consists of the state system (1) with its initial conditions, the adjoint system (4)–(5), and the characterization of the optimal controls (6). Since the adjoint system is linear in λ_A and λ_B with bounded coefficients, the solutions are bounded for all $t \in [0,T]$. There- fore, solutions of our optimality system are bounded for all $t \in [0,T]$. Such bounds give the existence and uniqueness of the optimality system for sufficiently small final time T together with bounded coefficients, initial conditions, and transversality conditions (Fister, 1998), which implies the uniqueness of the optimal control.

Parameter Estimation

The parameters for model (1) were estimated using annual landing data for the Black Sea anchovy (STECF, 2017; TUIK 2023) and the Atlantic bonito (FAO, 2017; TUIK 2023). The available data from 2000 to 2022 was used to reflect the current state of the stocks and the current harvesting practices. The Ordinary Least Squares (OLS) method was used to estimate the parameters by minimizing the sum of the squared differences between the observed landing data and the predictions made by the model (1).

The quality of the fit was evaluated by calculating the relative error using the following formula:

$$\min\left(\frac{\sum_{i=1}^{n}(A_i - \hat{A}_{est})^2}{\sum_{i=1}^{n}(A_i)^2} + \frac{\sum_{i=1}^{n}(B_i - \hat{B}_{est})^2}{\sum_{i=1}^{n}(B_i)^2}\right) \quad (8)$$

where A_i and $\hat{A}_{est}$ are the exact and estimated annual anchovy landings, and B_i and $\hat{B}_{est}$ are exact and estimated annual bonito landing, respectively. To estimate the model parameters and constant harvest rates of the fisheries, we used the ode45 solver with multistart and fmincon from the Optimization Toolbox of MATLAB. The stability condition $\alpha(1 + \delta) < 1$ for the coexistence equilibrium point was taken as a constraint in the parameter estimation to ensure sustainable harvesting rates. The estimated parameter values, except for h_1 and h_2 (we vary these rates), were used to produce all the numerical solutions and outcomes. We estimated the initial biomass of the anchovy and the bonito, 4 parameters (r, K, a, and b), and the constant harvest rates h_1 and h_2 using 23 years of landing data from 2000 to 2022, with the time unit in days.

Results

In this section, our main purpose is to estimate the optimal and sustainable amount of landings that should be made from the southern part of the Black Sea for the Black Sea anchovy and the Atlantic bonito. Another objective of this study is to examine the predator effect of the Atlantic bonito on the Black Sea anchovy population. Additionally, we aim to investigate the benefits of conducting optimal and sustainable landings in terms of their net profits and annual average landings over the course of 23 years.

According to a study by Daskalov et al. (2020), around %63 of the bonito's diet comes from preying on anchovy. With this finding, two preliminary investigations were conducted. Firstly, the landing data for both species were examined to determine if there was evidence of this predator effect. The left plot in Figure 1 indicated a significant sign of such an effect. Secondly, the flow diagram of the species (Figure 1) was examined to further confirm the predator effect on the anchovy population.

Following the preliminary investigations, we proceeded to examine and measure the present condition of fish stocks in section 3.1 including the influence of bonito as a predator on the anchovy population. Furthermore, in section 3.2, we explored an optimal harvesting strategy that would maximize the landing of these fish stocks.

Current Status of the Fisheries

In the southern region of the Black Sea, a harvesting approach has been implemented whereby constant harvest rates are applied each fishing season to both species, based on annual landing data. However, due to the scarcity of annual landing data, accurately estimating the current harvesting strategy is challenging. This is because the number of estimated parameters cannot be more than the number of data points in a well-posed parameter estimation. If this rule is not followed, the estimated parameters will not be unique, and the estimation will become ill-posed.

Therefore, to capture the current status of fish stocks, we followed a particular procedure. First, we used the parameters listed in Table 1 and maintained a constant value for h_2 as 0.22 throughout the bonito harvest seasons from 2000 to 2022. Then, we estimated the annual harvest rates for the anchovy fishery by fitting the annual anchovy landing data. We did this by making a separate constant estimate for each year instead of having one constant harvest rate for all the years, resulting in 23 different constant harvest rates for the anchovy fishery. Next, using the same parameters in Table 2, we kept the annual estimated constant harvest rates the same for the anchovy harvest and fit the bonito landing data to estimate the annual constant harvest rates for the bonito fishery. After obtaining all the annual harvest rates for both fisheries, we predicted the predator-prey plots, biomass, landings, consumed anchovy amount by bonito, and new bonito individuals due to consumption of the anchovy in Figures 2 and 3.

The application of this procedure yielded average harvest rates of $h_1 = 0.38$ and $h_2 = 0.23$ in the estimated current harvest strategy. It's worth noting that these values differ slightly from the harvest rates we obtained during our parameter estimation process. Our estimated current harvesting strategy involves the use of distinct harvest rates for individual years, and we then calculate their arithmetic mean to determine the average harvest rates. This approach allows for the possibility of fluctuations, as in some years, harvest rates can be considerably higher or lower than the calculated average.

According to the estimated current harvesting strategy, the Atlantic bonito is predicted to consume an average of 93,259 tonnes of anchovy annually, while the predicted annual landing of anchovy is about 201,030 tonnes. This indicates that a significant portion of the anchovy biomass, corresponds to 46% of the anchovy

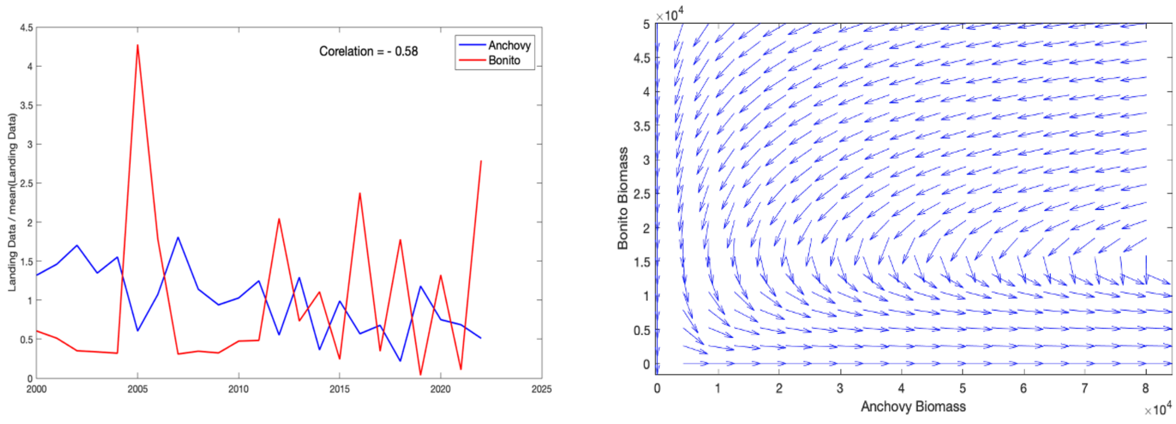

Figure 1. Left plot: Annual landing of the Atlantic bonito and the Black Sea anchovy over years when landing data normalized by its mean for both species. Right plot: The flow diagram of predator-prey relation between the Black Sea anchovy and the Atlantic bonito on the southern part of the Black Sea

Table 2. Comparison of harvesting strategies by assuming 50% net profit in the estimated current harvesting strategies of the Atlantic bonito and the Black Sea anchovy fisheries. The percentage of total net profit is calculated as total net profit over total cost The harvest rates in optimal and sustainable strategies are constant for each season

	Harvesting Strategies	Annual Average Harvest Rate	Annual Average Landing/Catch (Tonnes)	Total Cost (US Dollar)	Total Revenue (US Dollar)	Total Net Profit (US Dollar)	Total Net Profit (%)
Anchovy	Estimated Current Harvesting	0.38	201,030	606,830,000	910,340,000	303,510,000	50%
	Optimal and Sustainable Harvesting	0.34	210,570	518,380,000	841,820,000	323,440,000	62%
Bonito	Estimated Current Harvesting	0.23	16,192	182,61,000	274,040,000	91,429,000	50%
	Optimal and Sustainable Harvesting	0.22	16,632	169,380,000	270,050,000	100,670,000	59%

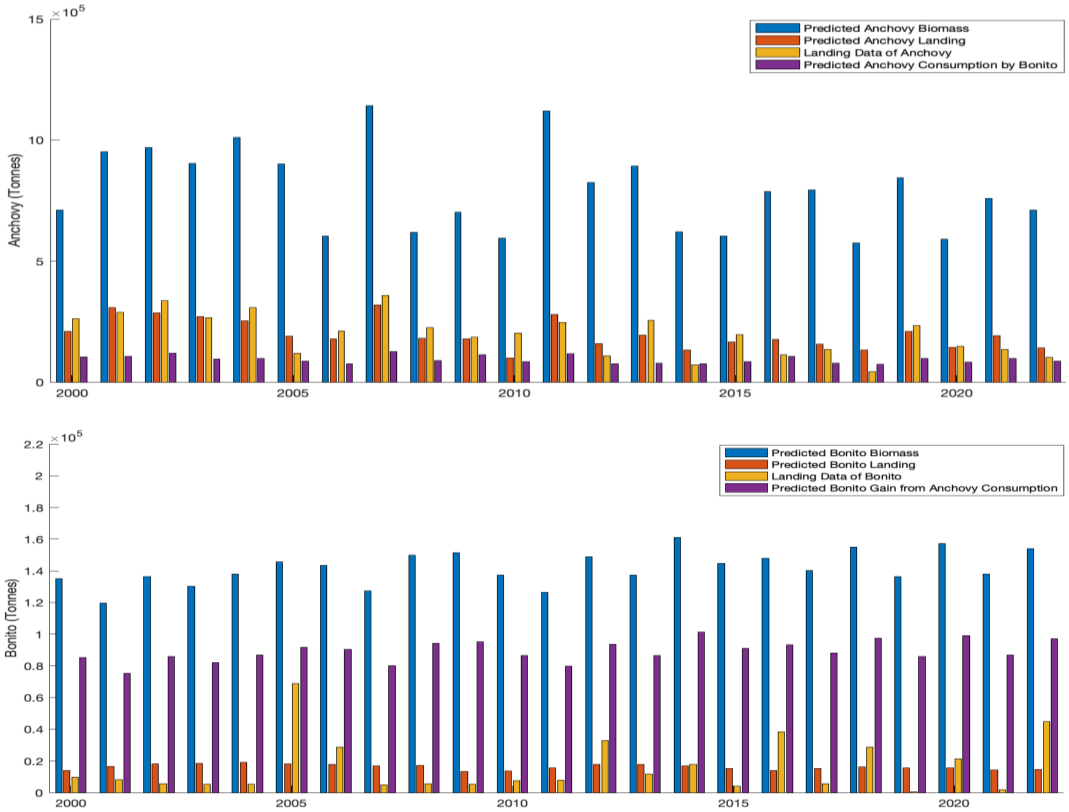

Figure 2. Top chart: Predicted anchovy biomass, predicted anchovy landing, landing data of anchovy, and predicted anchovy consumption by bonito. Bottom chart: Predicted bonito biomass, predicted bonito landing, landing data of bonito, and predicted bonito gain due to consumption of anchovy

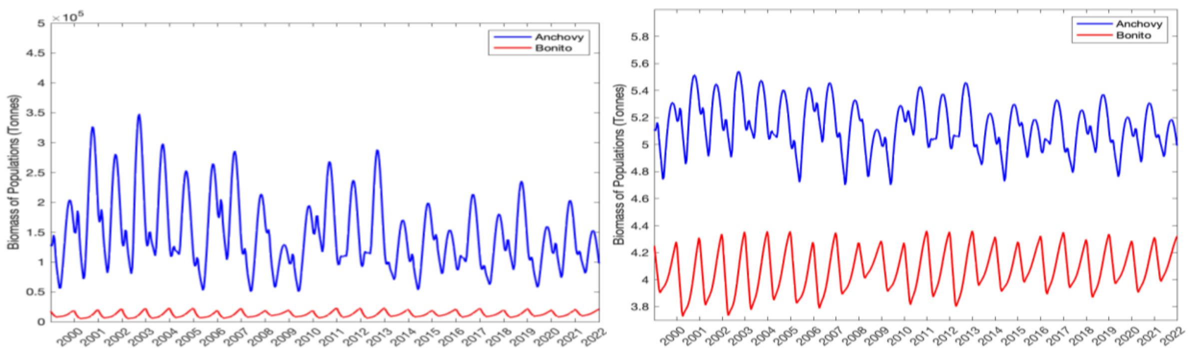

Figure 3. Left panel: Predicted predator-prey relation of anchovy and bonito. Right panel: Predicted predator prey relation of anchovy in log10 scale.

landing, is being consumed by the Atlantic bonito. In some years, the amount of anchovy consumption due to bonito predation exceeds the anchovy landing (as shown in the top plot of Figure 2).

Since this study assumes that 63% of the Atlantic bonito's diet is supplied by the Black Sea anchovy (Daskalov et al., 2020), our results depend on this assumption. Decreasing this percentage 10% results in 13% increase (the average landing changes from 201,030 tonnes to 226,970 tonnes) in the anchovy landing and 40% increase (the total net profit changes from 303,510,000 US Dollar to 426,280,000 US Dollar) in the anchovy profit in the estimated current harvesting case. On the other hand, increasing this percentage 10% causes in 8% decrease (the average landing changes from 201,030 tonnes to 185,450 tonnes) in the anchovy landing and 22% reduction (the total net profit changes from 303,510,000 US Dollar to 238,120,000 US Dollar) in the net profit of anchovy. These changes are very small in the Atlantic bonito fishery as we compare with the anchovy fishery (see Table 3).

Due to the presence of the extremely high landing (68,830 tonnes) in the year 2005 for the Atlantic bonito, the fishery model is unable to accurately capture it. Additionally, in order to capture this particular data point, the model fails to accurately capture low landing data points as well, resulting in an estimated landing value that falls between the high and low data points. On the other hand, the variation among the anchovy landing data is not as high, and therefore, the model is better able to capture the anchovy landing data points.

The analysis of annual landings and predicted stocks reveals that the harvesting of these fish populations is not consistent, which leads to significant fluctuations in both landings and biomass (as shown in Figures 2 and 3). To avoid such fluctuations and promote sustainable fishing practices, a consistent and optimal harvesting strategy is necessary. In the next section, I will discuss an optimal landing strategy using the optimal control approach outlined in section 2.3.

Status of the Fisheries in the Optimal Harvesting Strategy

The process of implementing optimal control involves utilizing the parameter values outlined in Table

Table 3. Comparison of harvesting strategies by assuming 50% net profit in the estimated current harvesting strategies of the Atlantic bonito and the Black Sea anchovy fisheries. The results are driven by different parentages of the Atlantic bonito diets coming from the Black Sea anchovy consumption

	Harvesting Strategies	Percentage of Bonito diets comes from anchovy consumption	Annual Average Harvest Rate	Annual Average Landing/Catch (Tonnes)	Total Cost (US Dollar)	Total Revenue (US Dollar)	Total Net Profit (US Dollar)	Total Net Profit (%)
Anchovy	Estimated Current Harvesting	63%	0.38	201,030	606,830,000	910,340,000	303,510,000	50%
		56.7% (%10 Below)	0.38	226,970	606,830,000	1,033,100,000	426,280,000	70%
		69.3% (10% Above)	0.38	185,450	606,830,000	844,940,000	238,120,00	39%
	Optimal and Sustainable Harvesting	63%	0.34	210,570	518,380,000	841,820,000	323,440,000	62%
		56.7% (10% Below)	0.34	235,020	518,380,000	932,800,000	414,410,000	80%
		69.3% (10% Above)	0.34	189,200	518,380,000	772,040,000	253,650,00	49%
Bonito	Estimated Current Harvesting	63%	0.23	16,192	182,61,000	274,040,000	91,429,000	50%
		56.7% (10% Below)	0.23	15,688	182,61,000	263,980,000	81,369,000	45%
		69.3% (10% Above)	0.23	16,415	182,61,000	277,590,000	94,983,000	52%
	Optimal and Sustainable Harvesting	63%	0.22	16,632	169,380,000	270,050,000	100,670,000	59%
		56.7% (10% Below)	0.22	16,177	169,380,000	260,560,000	91,186,000	54%
		69.3% (10% Above)	0.22	16,963	169,380,000	272,580,000	103,200,000	61%

1 and integrating several essential components into the system. These elements consist of state variables (as described by Equation 1), adjoint variables (specified in Equation 4), transversality conditions (as outlined in Equation 5), and characterizations of optimal controls (as defined by Equation 6). In the characterization of optimal controls, we set the maximum harvest rates for the anchovy fishery as M = 0.36 and for the bonito fishery as M = 0.22. These values are obtained from the constant harvest rate estimation to ensure both a stable predator-prey system and sustainable fishing practices in the long term for the southern part of the Black Sea.

The results of the optimal control strategy show that it is very similar to applying a constant harvest rate for each fishing season, as seen in Figure 4. The optimal control suggests harvesting the anchovy at a rate of $h_1 = 0.34$ and the bonito at a rate of $h_2 = 0.22$. We obtained these values by varying the upper bound in optimal control case to obtain the best landing and profits for the both fisheries. The fact that the optimal control suggests constant harvest rates may be due to the short time period considered in the fisheries or the low maximum harvest rate obtained from the stability analysis of these species. Further investigations are required to understand why the optimal control strategy chooses these constant harvest rates, but this is beyond the scope of this study.

The optimal control strategy developed for these two species offers a more sustainable and consistent approach to managing the predator-prey relationship and landings, as demonstrated in Figures 4 and 5 when compared to the current status of the harvesting strategy. Additionally, the optimal control strategy results in higher landings and profits for both species, as shown in Table 2. Fishing quotas based on the average annual landing of bonito and anchovy can be implemented to ensure optimal and sustainable fisheries for both species.

In the optimal harvesting strategy, the predicted harvest rates, landings, fishing costs, revenues, and net profits are compared with the current status of the fishery presented in section 3.1 in Table 2. The comparison is conducted with the assumption of a 50% net profit in both bonito and anchovy fisheries, based on the estimated current harvesting strategy. To achieve this profit margin, the coefficients, $\mu_1, \mu_2, \mu_3, \mu_4$, and α, which are associated with the cost of fisheries, have been adjusted accordingly. (as provided in Table 2). Note that the percentage of the total net profit is calculated as total net profit over total cost in Tables 2 and 3. The results indicate that the optimal and sustainable strategy provides higher landings and net profits for both fisheries compared to the current harvesting strategy. The optimal control strategy provides 24% and 18% higher profits for anchovy and bonito, respectively. Additionally, it results in a 5% increase in landings for anchovy and a 3% increase in landings for bonito (see Table 2). Moreover, the cost of landing for anchovy is lower in the optimal strategy as compared to the current harvesting strategy due to the estimating low harvest rate in the optimal strategy to maintain stable predator-prey dynamics.

Discussion

Incorporating predator-prey dynamics into fishery management is crucial for ensuring sustainable and optimal harvests. As demonstrated in this study, the predator effect of bonito on the anchovy population can cause significant reductions in anchovy biomass, even more so than the direct impact of anchovy fishing (see Figure 2). Ignoring this predator effect in a single-species model for anchovy would result in an overestimation of anchovy biomass and potential overfishing, leading to the collapse of the fishery. Therefore, taking into account the interactions between predator and prey populations is important in developing effective and sustainable fisheries management strategies. This study also has the advantage of incorporating stability analysis for the predator-prey system, which ensures a stable and sustainable predator-prey relationship between the two species. This provides a basis for sustainable fishing practices and ultimately leads to increased profits (Table 2).

In addition to the benefits of considering predator-prey relationships and stability analysis, the use of optimal control theory to optimize sustainable landing also brings significant advantages. As shown in Figures 3 and 4, the predator-prey relationship is stabilized and optimized by utilizing the tools of stability analysis and optimal control in the fishery model. This approach helps prevent overfishing and stabilizes population biomass and landings. Furthermore, it leads to increased profits and landings compared to the predicted results from the current status of fisheries in section 3.1. The optimal control strategy leads to profits that are 24% higher for anchovy and 18% higher for bonito, respectively. Moreover, it leads to a 5% increase in anchovy landings and a 3% increase in bonito landings (see Table 2).

In our study, we made a choice to exclude the consideration of logistic growth for Atlantic bonito, a decision driven by three primary factors. Firstly, Atlantic bonito ranks among the top predators inhabiting the southern portion of the Black Sea, leading us to anticipate minimal predation effects on its population dynamics. Second, the data, drawing from the findings of Daskalov et al. (2020), revealed that a substantial 63% of the Atlantic bonito's diet is attributed to anchovy consumption. This information allowed us to incorporate the remaining 37% contribution to the growth of bonito. Finally, we aimed to streamline our modeling efforts by reducing the number of estimated parameters by 2. Additionally, we sought to investigate the sensitivity of these fisheries to changes in the percentage representing bonito's predation impact on anchovy populations.

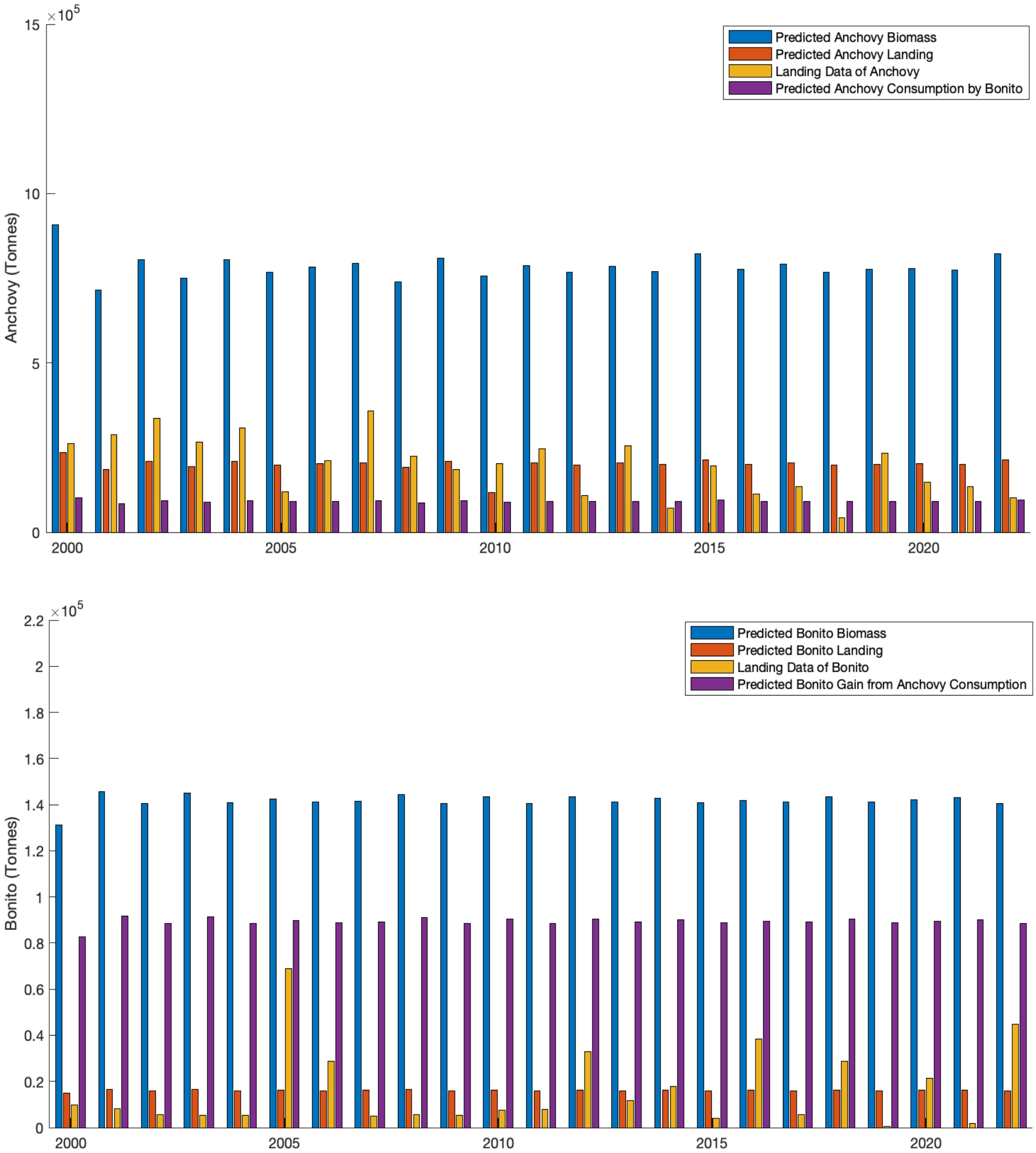

Figure 4. Top chart: Predicted anchovy biomass, predicted anchovy landing, landing data of anchovy, and predicted anchovy consumption by bonito in the optimal control strategy. Bottom chart: Predicted bonito biomass, predicted bonito landing, landing data of bonito, and predicted bonito gain due to consumption of anchovy in the optimal control strategy.

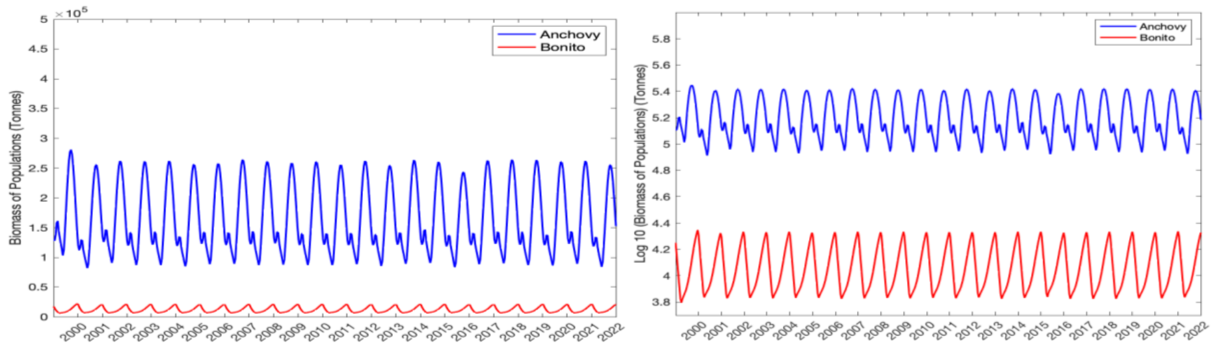

Figure 5. *Left panel: Predicted predator-prey relation of anchovy and bonito in* the optimal control strategy. Right panel: *Predicted predator-prey relation of anchovy in log10 scale in* the optimal control strategy

Our investigation showed that the assumption of 63% is very sensitive especially for the anchovy fishery since 10% changes in this percentage causes annually 13% changes in the anchovy landing and 40% changes in the net profit of anchovy. We did not see dramatic changes in the Atlantic bonito fishery as we saw in the anchovy fishery since 10% changes in this percentage only results in up to 3% changes in the bonito landing and up to 10% changes in the net profit of bonito (see Table 3).

When comparing our model's output with the literature, we observe that the current average harvest rate for bonito and anchovy closely aligns with the studies presented by Salihoglu, Arkin, Akoglu, and Fach (2017). They estimated the Atlantic bonito and anchovy harvest rates as 0.26 and 0.41, respectively, between the years 2000 and 2014, while our estimates between the years 2000 and 2022 were 0.23 and 0.38 (see Table 2). Additionally, our maximum harvest rate for the anchovy fishery, which was 0.34 in the optimal control case, closely mirrors the findings presented by Demir and Lenhart in 2019, where they reported a maximum harvest rate of 0.35 between years 2002 and 2016.

The MSY (Maximum Sustainable Yield) for Atlantic bonito was estimated at approximately 17,000 tonnes, with a range of 14,700 to 19,800 tonnes, according to the study by Daskalov et al. in 2020. It's worth noting that our calculated optimal and sustainable yield for Atlantic bonito, which stands at 16,632 tonnes, closely aligns with this result. Similarly, we see that the optimal annual landing estimated as 222,250 tonnes in Demir and Lenhart (2019) and this result in the range of our estimates since our estimates is 210,570 with the range of 189,200 and 235,020 when we vary the percentage of bonito diets on the anchovy consumption (see Table 3).

We have highlighted that the anchovy landing is more than twice as high as the predation mortality of anchovy since we showed that the consumption of anchovy by bonito corresponds to 46% of annual average landing of anchovy. This observation aligns with the findings presented by Daskalov et al. in 2020. Their research demonstrated that the fishing mortality of anchovy (the anchovy landing) has been consistently two to three times greater than the predation mortality inflicted by bonito on anchovy between years 1995 and 2015.

Data Limitation and Model Selection

Given the absence of direct measurements for fish stocks, we rely on alternative data sources that provide information about populations, such as landing data and catch per unit effort (CPUE) data. These datasets enable us to monitor changes in population abundance. Moreover, by utilizing such data and fitting them into models, we can derive other critical insights, including harvest rates, maximum sustainable yield, population abundance estimates, and more. The selection of an appropriate model is crucial, depending on the specific objectives of a study, to obtain targeted information about fish stocks.

In our current study, our primary objectives include investigating the abundance of the bonito and anchovy populations, quantifying optimal and sustainable yields, and exploring the predator effect of bonito on anchovy. To achieve these goals, we have employed a predator-prey model and have analytically implemented both stability and optimal control tools.

In comparison to other traditional stock assessment methods like XSA, VPA, BMS, and CMSY, our modeling approach offers several advantages. First and foremost, our model requires less data. For instance, in this study, we utilized landing data, annual fish prices per kilogram, and diet data to derive significant insights into fish stocks. Conversely, if any of these traditional stock assessment methods were employed, additional data, such as catch per unit effort (CPUE) data, estimates of natural mortality, predator ratios, and more, would be necessary.

Furthermore, our choice of model enables us to conduct stability and optimal control analyses both analytically and numerically. In contrast, conducting such analyses within traditional stock assessment methods can be challenging, especially when dealing with multiple trophic levels and species. Even though some traditional stock assessment methods offer stability analysis for single-species models, they often lack the necessary tools to implement optimal controls effectively. Hence, we have opted for a straightforward predator-prey model and subsequently applied stability and optimal control tools, aligning with our research objectives.

Certainly, there are some limitations associated with our model selection, primarily because we employ a deterministic model. Deterministic modeling approaches offer average predictions for each time step, overlooking the inherent natural variations in population biomass caused by factors such as birth rates, migration patterns, and temperature fluctuations. Consequently, the model struggles to accurately represent exceedingly high or low landings, as evident in Figure 2. While the model performs adequately for landing data within moderate ranges, it falls short when attempting to capture the sharp fluctuations in landing data, particularly for the Atlantic bonito.

To enhance the model's accuracy, one potential solution is to introduce a noise term into the model, although this may still prove insufficient in capturing extreme landing data points. Therefore, an alternative approach involves smoothing the landing data to reduce noise prior to model fitting. This strategy can enhance the model's performance by minimizing both process and measurement errors.

The study focused solely on the consumptive effect of bonito on the anchovy population, which involves reducing the abundance of prey. However, predators can also influence prey populations through non-consumptive effects (Marino et. al., 2019), such as changing their foraging behavior and migration patterns. For example, the study could consider bonito's impact on anchovy's migration patch in the southern part of the Black Sea. Accounting for these non-consumptive effects may lead to an improvement in the model's performance.

Conclusion

In summary, the study highlights the importance of optimizing sustainable landing via optimal control theory for the Atlantic bonito and anchovy fish stocks. The current status of these stocks is inconsistent, leading to unpredictable landings and profits. Optimizing sustainable harvesting not only provides more landings and profits but also ensures a sustainable and consistent predator-prey relationship between the two species. The average landings obtained through optimal control strategy can be used as fishing quotas to achieve optimal and sustainable fisheries for these species.

The findings demonstrate that adopting the optimal and sustainable strategy yields greater landings and net profits for both fisheries when contrasted with the current harvesting approach. Specifically, the optimal control strategy leads to a 24% increase in profits for anchovy and an 18% increase for bonito. Moreover, it generates a 5% rise in anchovy landings and a 3% increase in bonito landings.

Our investigation has highlighted the significant impact of the Atlantic bonito's feeding habits on the anchovy fishery, underscoring its critical role in these fisheries, particularly the anchovy fishery. Our findings reveal that annually, a substantial 93,259 tonnes of anchovy are consumed by the Atlantic bonito, equivalent to 46% of the annual anchovy landing. Consequently, incorporating predator-prey relationships into fishery models can enhance the realism of the outputs. Notably, our results closely corroborate existing literature as mentioned in the discussion section.

This study also recommends a holistic approach to fishery management by including predator-prey relations in fishery models, implementing stability analyses to achieve sustainable fishery, and optimizing sustainable landing via optimal control theory. It also highlights the advantage of this modeling approach, which requires less data than other stock assessment methods. Overall, the study proposes a more realistic and sustainable management strategy for harvesting fish stocks.

Supplementary Materials

MATLAB codes and data can be shared on reasonable request to the corresponding author.

Ethical Statement

Local Ethics Committee Approval was not obtained because experimental animals were not used.

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Conflict of Interest

The author declares that no financial interests or personal relationships may affect this work.

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Appendix

Details for Stability Section 2.2

To simplify our stability analysis, we first reduced the number of parameters from 6 to 3 by rescaling the Equation (1). We first let $x = \frac{A}{K}$ (*A=Kx*) and obtained $\frac{dA}{dT} = K\,\frac{dx}{dT}$. Then, substitute these in Equation (1) as

$$K\frac{dx}{dT} = rxK(1-x) - axKB - h_1 xK$$

$$\frac{dB}{dt} = bxKB - h_2 B$$

After that we simplified the anchovy equation by removing K from both sides, then let y=$\frac{a}{r}B$ $\quad(B = \frac{r}{a}$ *y*) and $\frac{dy}{dt} = \frac{a}{r}\frac{dB}{T}$. When we substitute them in above equation, then will obtain the following equation

$$\frac{dx}{dT} = rx(1-x) - ax\,\frac{r}{a}\,\mathrm{y} - h_1 x$$

$$\frac{r}{a}\frac{dy}{dT} = bxK\frac{r}{a}y - h_2\frac{r}{a}\,\mathrm{y}$$

When we simplify the above equation and remove $\frac{r}{a}$ from the bottom equation, we will get the following equation by letting $t = rT$

$$r\frac{dx}{dt} = rx(1-x) - rx\mathrm{y} - h_1 x$$

$$r\frac{dy}{dt} = \ bKxy - h_2\mathrm{y}$$

we now simplify the parameter r and arranged the above equation as

$$\frac{dx}{dt} = x[1 - x - y - \frac{h_1}{r}x]$$

$$\frac{dy}{dt} = \frac{bK}{r}y[x - \frac{h_2}{bK}]$$

When we let $\beta = \frac{bK}{r}$, $\alpha = \frac{h_2}{bK}$ and $\delta = \frac{h_1}{r}$, we obtain the equation 2 as follow

$$\frac{dx}{dt} = (1 - y - (1 + \delta)x)$$

$$\frac{dy}{dt} = \beta y(x - a).$$

Data Used in This Study

The annual landing data for the Black Sea anchovy and the Atlantic bonito (FAO, 2017; TUIK, 2023) in the southern part of the Black Sea, the annual average anchovy price (TUIK, 2023), and the annual average exchange rate from Turkish Liras to US Dollar (Turkish Central Bank, 2023) are presented in Table 4. These are the data used in this study, as reported in the article.

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