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RESEARCH PAPER

Heterogeneous Growth Prediction in Farmed Tilapia

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Abstract

This study displays the application of the quantile regression theory to predict the size heterogeneity of cultured organisms. The analysis was applied to empirical data of the tilapia cultured in freshwater. Tilapia was cultured at four diets (50%, 80%, 100%, and Satiation). The quantile regression (QR) demonstrated to successfully model the size heterogeneity in tilapia ($p<0{,}05$; $u<0{,}20$), due to the feeding strategies effect. These results indicate tilapia fed an 80% ration size simulated a maximum biomass of 2,345.17 and 2,853.38kg at the harvest size of 200-300g (at 180 days) and 300-400g (at 210 days). The simulation of the quantile curves at a higher production scale allowed an estimate of the biomass distribution according to different market sizes, this strengthens management decision making in tilapia aquaculture. Implications of quantile regression and size heterogeneity in aquaculture are presented here.

Introduction

Size variability is a common phenomenon in cultured organisms since individuals are prone to differences in size, weight, and growth rates (Peacor *et al*., 2007). This is mainly due to the hierarchical behaviour of the species, the effects of high densities and the rationing size that might cause competition (Barbosa *et al*., 2006; Kestemont *et al.,* 2003; Potthoff & Christman, 2006). Size heterogeneity is caused by shifts in individual growth rates at the initial culture stage, stabilizing after the organisms reach a specific size (Peacor *et al*., 2007). This can affect the optimal management of populations, by modifying the population's distribution through time and thus negatively impacting the economy of the farm (Borrego-Kim *et al*., 2020a,b; Gasca-Leyva *et al*., 2008; Vicenzi *et al*., 2014).

In aquaculture, the problem of size heterogeneity has been studied in species several (Araneda *et al*., 2018; Borrego-Kim *et al*., 2020a,b; Estruch *et al*., 2017; Gasca-Leyva *et al*., 2008; Mayer *et al*., 2009;). Different methods have been proposed to analyse size heterogeneity in aquaculture, for instance, the variance simulation through an initial distribution (Summerfelt et al., 1993), the modelling of a population structured by size (Arnason *et al*., 1992; Gasca-Leyva *et al*., 2008; Peacor *et al*., 2007), weighing by inverse size to minimize the effect of the increase in the size variance (Santos *et al.,* 2008), and determine the growth asymmetry (the evolution of heterogeneity through time) with the Bawling index (Borrego-Kim *et al*., 2020a). A stochastic growth model has also been proposed (Yoshioka & Yaegashi, 2017). Briceño *et al*. (2010) applied generalized linear models (GLMs) to analyse the broad growth variability in individual sizes of the octopus

Octopus maya in experimental systems. Araneda *et al*. (2013) modelled the growing heterogeneity of the Pacific white shrimp cultured in freshwater by considering the effect of initial density within the culture and via the Peacor *et al*. (2007) theory. In Japan, Yoshioka and Yaegashi (2017) proposed a stochastic growth model for the culture of *Plecoglossus altivelis* for educational purposes (not for the market).

Different contributions of quantile regressions (QR) have been developed in aquaculture for different purposes (Brazenor & Hutson, 2015; Bogard *et al*., 2017; Jacobsen, 2017; Steen & Jacobsen, 2020). However, few applied studies model the growth path of cultured organisms through time despite the efforts in the use of QR in aquaculture. In fact, only Mayer et al. (2009), Jover and Estruch (2017) and Estruch *et al*. (2017) have applied (QR) to study the growth variation of the gilt-head bream through time. And these authors modelled growth data from farms taking into account the effect of water temperature. Furthermore, given size heterogeneity in aquaculture is a constant problem faced by producers (Barbosa *et al*., 2006; Kestemont *et al*., 2003; Potthoff & Christman, 2006). Tilapia producers receive pressure from the market and financial conditions to generate uniform product sizes (Azaza *et al*., 2013; Borrego-Kim *et al*., 2020a,b; Khaw *et al*., 2016). This study aims to analyse the size dispersion throughout quantile regression applied to biological growth data of the tilapia cultured.

Materials and Methods

Data Source

Growth data from tilapia was obtained from the Centre for Research and Advanced Studies (CINVESTAV) at the aquaculture facilities of Merida Unit, Mexico, data was collected from an experimental system of tilapia fattening culture cycle. Four treatments were tested, three following the recommended ration by feeding tables:100, 80, 50 (% body weight/day) (by duplicate), and a Satiation rationing (>110 (% body weight/day) of the feeding table). Treatments were normalized under the [0,1] range. The experimental system consisted of eight circular fiberglass tanks placed indoors with a volume of 0.75m^3 of useful capacity per tank through a recirculating semi-closed system and waste trap and constant aeration. Initial stocking density was 44.0 fish /m^3 with 14 g per fish and biometrics were carried out every 14 days; more culture details are found in Poot-López *et al*. (2014).

Growth Model

Tilapia growth has been represented by various exponential, logarithmic and asymptotic models, respectively (Ansah & Frimpong, 2015; Dumas *et al.,* 2010; Rosa *et al*., 1997; Santos *et al.,* 2013). In this case, Von Bertalanffy (vB) (eq. 1) model was used considering the suggestion by Cai *et al*. (2018) and Vicenzi *et al*. (2014) on modelling with higher simplicity in order to improve data interpretation. Thus, the individual weights of the organisms through the quantiles were given by the following relationship:

$$W_i(t) = W_{\infty i}\left(1 - b_i e^{-k_i(t-t_{0i})}\right)^3 \quad (1)$$

Where, W_i represents the organism's weight for each quantile through time, $W_{\infty i}$ represents the infinite weight of the organism for each quantile, b_i is a constant, e represents the natural logarithmic base, k_i represents the instant growth rate for each quantile per day, t_{0i} represents the theoretical time for each quantile in which the organism's weight is zero.

It has been considered that the total population of organisms in the culture system $\left(N_Q(t)\right)$ is the sum of the number of individuals for each quantile and is given by:

$$N_Q(t) = \sum_{i=1}^{n} N_i(t) \quad (2)$$

Where n represents the number of quantiles and $N_i(t)$ represents the number of fish for each quantile over time and is affected by the same constant mortality rate (μ) over time and is given by:

$$N_i(t) = N_{0i}e^{-\mu t} \quad (3)$$

Where N_{0i} represents the number of individuals initially seeded for each quantile. And the total number of organisms initially seeded N_0 is given by:

$$N_0 = \sum_{i=1}^{n} N_{0i}$$

Therefore, the total biomass of the crop system at time $\left(B_Q(t)\right)$, was determined by the sum of the biomasses for each quantile i, which is given by:

$$B_Q(t) = \sum_{i=1}^{n} B_i(t) \quad (4)$$

Where $B_i(t)$ represents the biomass in each quantile, which was determined by multiplying the weight of the organism for each quantile by the number of individuals in each quantile i, as follows:

$$B_i(t) = W_i(t)N_i(t) \quad (5)$$

Quantile Regression

Estimate the parameters by the least square method aims to minimize the square sum of residuals, whereas quantile regression (QR) aims to minimize the absolute value of the weighted sum of the errors (Koenker & Basset, 1978; Koenker & Mizera, 2004). Where *X* is a real random variable characterized by a distribution function F(x) as indicated below:

$$F_X(x) = P(X \le x) \quad (6)$$

Where τ –th quantile of *X* is defined by $F^{-1}(\tau) = \inf\{x : \tau \le F(X)\}$ with $\tau \in (0, 1)$. The quantile function provides a characterization of the random variable of *X* interest. Thus, quantile curves can be provided by an optimization problem, to find a *x* value of the random variable such as $F_X(x) = \tau$, is considered a function of asymmetric linear loss defined by:

$$\rho_\tau(u) = u(\tau - I(u < 0)) \text{ con } \tau \in (0, 1) \quad (7)$$

Where $I\,()$ denotates the indicator function. In eq. (4), τ is the quantile and $I\,(u<0)$ is equal to 1 if $u<0$, is true and 0 if false. The τ quantile defines the fraction of all observations that tend to be under the curve. Therefore, the eq. (2) represents the positive weighting of residuals ($u>0$, with weighting equal to τ) ($u>0$, with weighting equal to τ-1). If $\tau = 0.5$, half of the observations are below and the other half are above the curve, both have the same weighting $(\tau = \tau - 1)$, whereas the objective function (eq. 5) is simplified as

$$\frac{1}{2}\sum_{i=1}^{n} \rho_\tau |x_i - \xi|.$$

According to Koenker and Bassett (1978), quantile regression is an extension of the quantiles of the regression of absolute minimum deviation, which fits a function for a median individual. This problem is turned into an optimization problem (Koenker & Basset, 1982; Koenker & Hallock, 2001), as indicated below:

$$\underset{\xi \in \mathbb{R}}{Min} \sum_{i=1}^{n} \rho_\tau (x_i - \xi) \quad (8)$$

Where ξ is the solution in size (g) given by the model and x_i represents the i-th observed size (g) for tilapia. For further details about the theoretical approaches of QR we recommend consulting Koenker and Basset (1978), Koenker and Hallock (2001) and Koenker and Mizera (2004). If the τ value is high (closet to 1), the QR function fits better to the high values within the sampling (for instance, 0.95); if the τ value is low (close to 0) the QR function fits better to low values withing the sampling (for instance, 0.5).

Parameterization and Statistical Validation

Growth models were evaluated according to the significance degree of the parameters (p<0.05), standard error (SE), *t* and *p*-value from the statistical adjustment. The QR non-linear analysis was carried out through the R statistical software (Research Development Core Team, 2019) using the "quantreg" library (Koenker, 2005). The validation of curve quantiles simulation was through the root mean square error (RMSE), the IF index similar to R^2 (Rosa *et al.*, 1997) and the uncertainty coefficient of Theil (U) (Pindyck & Rubinstein, 1981; Power, 1993). The latter's value is found between the range of 0 and 1, indicating a perfect match between real data (observed quantile) and simulated data (quantile curve) if it's equal to zero.

The squared error consists of three indicators which are represented by the error ratio due to the bias of the U_M mean, U_V variance and U_C covariance (Pindyck & Rubinstein, 1981). Values of U<0.20 are considered acceptable models (Power, 1993). Additionally, the Akaike information criteria (AIC) (Anderson, 2008; Ansah & Frimpong, 2015) was used to measure the goodness of fit of a quantile model. The AIC describes the relationship between bias and variance within a quantile model. Low AIC values indicate a better data representation. Simulation of the biomass performance at the economic level of different harvest sizes was executed in Microsoft Excel 365.

Data and Assumptions from the Simulation Analysis

Quantile growth curves designated from the QR analysis (0.05, 0.10, 0.15, 0.25, 0.50, 0.75, 0.85 and 0.95 quantiles) were employed to simulate the total biomass of an aquaculture production cycle (one tank) was predicted using harvest strategies of 120, 150, 180, 210 and 240 days. Harvest size (x_h) was considered based on local and national market from Mexico (SNIIM, 2020; Poot-López et al., 2014); for tilapia the ranges of $x < 200$, $200 \le x < 300$, $300 \le x < 400$, $400 \le x < 500$ and $500g \le x$ were considered. The performance of the analysis was based on technical-biological assumptions (Table 1).

Results

von Bertalanffy Model

The non-linear QR analysis showed statistically significant results on the estimate of tilapia heterogeneous growth ($p<0.05$). Table 2 reflects the tested parameters of the vB model for each quantile. Predictions of each quantile model demonstrated that heterogeneous growth increased as the ration size as well. The analysis of non-linear QR at ration size of 50% predicted the higher asymptotic weights (1,923.41-912.20g) from all quantiles 0.05-0. 95, in comparison to ration size at 80%, 100% and Satiation. Predicted growth rates (*k*) were less at quantile 0.05 in comparison to quantile 0.95 in each of the evaluated ration sizes (Table 2). Figure 1, shows a successful representation of the growth variation observed based on the effect of size ration size through quantiles. For instance, quantiles 0.05 and 0.95 in ration size 50% had at the 180 days a size variation range between 161.69 and 257.57g, whereas in Satiation, the size variation ranged from 239. 67 to 461.09g.

Validation of Growth Models

Simulation statistics showed that the vB model successfully represents the heterogenous growth predictions of tilapia. Results are shown in Table 3 for each rationing treatment for tilapia. The von Bertalanffy model's root mean square error (RMSE) predicted values of 3.31-13.22g, IF in 97.62-99.57% and an AIC in

Table 1. Technical-biological and management parameters of tilapia farmed systems

Parameter	Concept	Unity	Value	Source
	Tank volume	m^3	267.00	Poot-López et *al*., (2014)
μ_T	Daily mortality rate	Tilapia/day	0.00044	Empirical estimate (average)
N_{0T}	Initial number of individuals homogeneous case	Tilapia	11000	Local company
x_{0T}	Stock weight of tilapia	g	14	Local company

Table 2. Results of non-linear quantile regression of the empirical data growth of tilapia fed at different ration sizes (50, 80, 100% and Satiation). W_∞ is the asymptotic weight of the fish, *k* is the growth rate and t_0 is theoretical time in which the fish has zero weight. SE and *t* represent the standard error and the t-value statistical adjustment, respectively

	Parameters								
Quantile (τ)	W_∞	$SE(W_\infty)$	$t(W_\infty)$	k	$SE(k)$	$t(k)$	t_0	$SE(t_0)$	$t(t_0)$
r=50%									
0.05	1,923.41	2,353.29	0.82	0.0021	0.0013	1.70	-94.69	14.90	-6.35
0.15	950.60	353.47	2.69	0.00033	0.0007	4.79	-82.15	6.43	-12.77
0.25	1,117.78	337.55	3.31	0.00031	0.0005	6.12	-80.52	4.85	-16.61
0.5	1,481.51	356.32	4.16	0.0028	0.0004	7.92	-81.55	3.54	-23.03
0.75	1,305.11	363.09	3.59	0.0032	0.0005	6.48	-79.52	4.38	-18.15
0.85	989.61	385.84	2.56	0.0038	0.0009	4.49	-74.04	5.54	-13.37
0.95	912.20	246.85	3.70	0.0043	0.0007	5.87	-68.20	4.70	-14.53
r=80%									
0.05	840.29	909.21	0.92	0.0041	0.0030	1.35	-67.43	30.47	-2.21
0.15	463.35	119.52	3.88	0.0072	0.0015	4.73	-47.60	7.65	-6.22
0.25	553.87	86.30	6.42	0.0068	0.0008	8.44	-47.49	3.71	-12.80
0.5	598.12	63.97	9.35	0.0070	0.0006	12.57	-45.77	2.40	-19.07
0.75	647.73	58.27	11.12	0.0071	0.0005	14.61	-45.81	1.68	-27.27
0.85	635.80	41.43	15.35	0.0075	0.0004	18.73	-43.91	1.39	-31.62
0.95	575.98	35.88	16.05	0.0085	0.0005	16.61	-40.92	1.51	-27.08
r=100%									
0.05	362.82	80.34	4.52	0.0071	0.0015	4.83	-57.96	9.14	-6.34
0.15	450.36	48.02	9.38	0.0079	0.0007	11.23	-44.36	2.57	-17.23
0.25	465.49	41.19	11.30	0.0082	0.0006	12.93	-42.42	2.30	-18.47
0.5	497.99	31.80	15.66	0.0091	0.0005	17.53	-37.24	1.65	-22.61
0.75	565.33	42.83	13.20	0.0092	0.0006	15.12	-36.80	1.50	-24.57
0.85	599.87	40.37	14.86	0.0093	0.0005	17.25	-36.48	1.33	-27.48
0.95	665.89	28.37	23.47	0.0091	0.0004	25.55	-36.03	0.96	-37.51
Satiation									
0.05	351.47	81.39	4.32	0.0100	0.0025	3.95	-32.18	10.20	-3.15
0.15	444.48	46.40	9.58	0.0089	0.0009	9.58	-38.93	3.71	-10.48
0.25	449.74	37.49	12.00	0.0092	0.0007	12.87	-38.80	2.36	-16.47
0.5	482.30	33.78	14.28	0.0097	0.0006	16.27	36.69	1.53	-24.04
0.75	582.68	72.71	8.01	0.0095	0.0011	8.88	-36.25	2.63	-13.80
0.85	618.67	68.23	9.07	0.0099	0.0009	10.73	-33.89	1.88	-18.04
0.95	649.70	52.48	12.38	0.0105	0.0008	12.96	-31.95	1.61	19.83

19.64-36.49 for tilapia growth data. The uncertainty coefficient of Theil showed that in vB model can adequately recreate the growth simulation in the analysed specie. Quantile curves had values of *U<0.2* (Power, 1993) for each rationing treatment.

Simulation of Production to a Commercial Production Cycle

Tilapia Study Case

Tilapia biomass from a production cycle was simulated based on quantile curves that considered a commercial tank and assumptions from Table 1. This analysis offers a harvest alternative for producers, as it benefits according to the size demands of the market. In Figure 2 the simulated biomass distribution is shown between 120 and 240 days according to different sizes. The harvest size range of <200, 200-300g and 300-400g were predominant in individuals fed with 50% and 80% rationings among the harvest range of 210 to 240 days, respectively. Tilapia fed with 50% rationing showed a biomass peak of 2,173.64 and 2,240.15kg on the harvest size of 200-300 g (210 days) and 300-400g (240 days). Similarly, fish fed with 80% rationing had biomass ranging from 2,345.17 and 2,853.38 kg on the harvest size of 200-300g (180 days) and 300-400g (210 days).

On the other hand, the 100% and Satiation rationings ranging from 200-300g, 300-400g and 400-500g had higher harvest frequency between 210 and 240 days, respectively. Tilapia fed with 100% rationing showed a biomass peak of 2,087.73 and 1,889.06kg on the harvest size of 300-400g (210 days) and 400-500g (240 days), respectively. The Satiation rationing had a biomass between 1,799.18 and 1,772.43kg on the harvest size of 200-300g (180 days) and 300-400g (210 days), respectively. Harvest size higher than 500g was found in tilapias fed with 100% and Satiation rationings during harvest days of 210 and 240, respectively.

Discussion

Growth Model

vB successfully modeled the pattern changes in the size of tilapia through quantiles. Similar results were reported in studies carried out for the gilt-head bream, where the effect of temperature was considered, although these authors used an exponential model (Estruch *et al*., 2017; Jover & Estruch, 2017; Mayer *et al*., 2009). The growth representation of cultured organisms by mean models based on regressions that are single or multiple, non-linear or linear, is not sufficient. This could

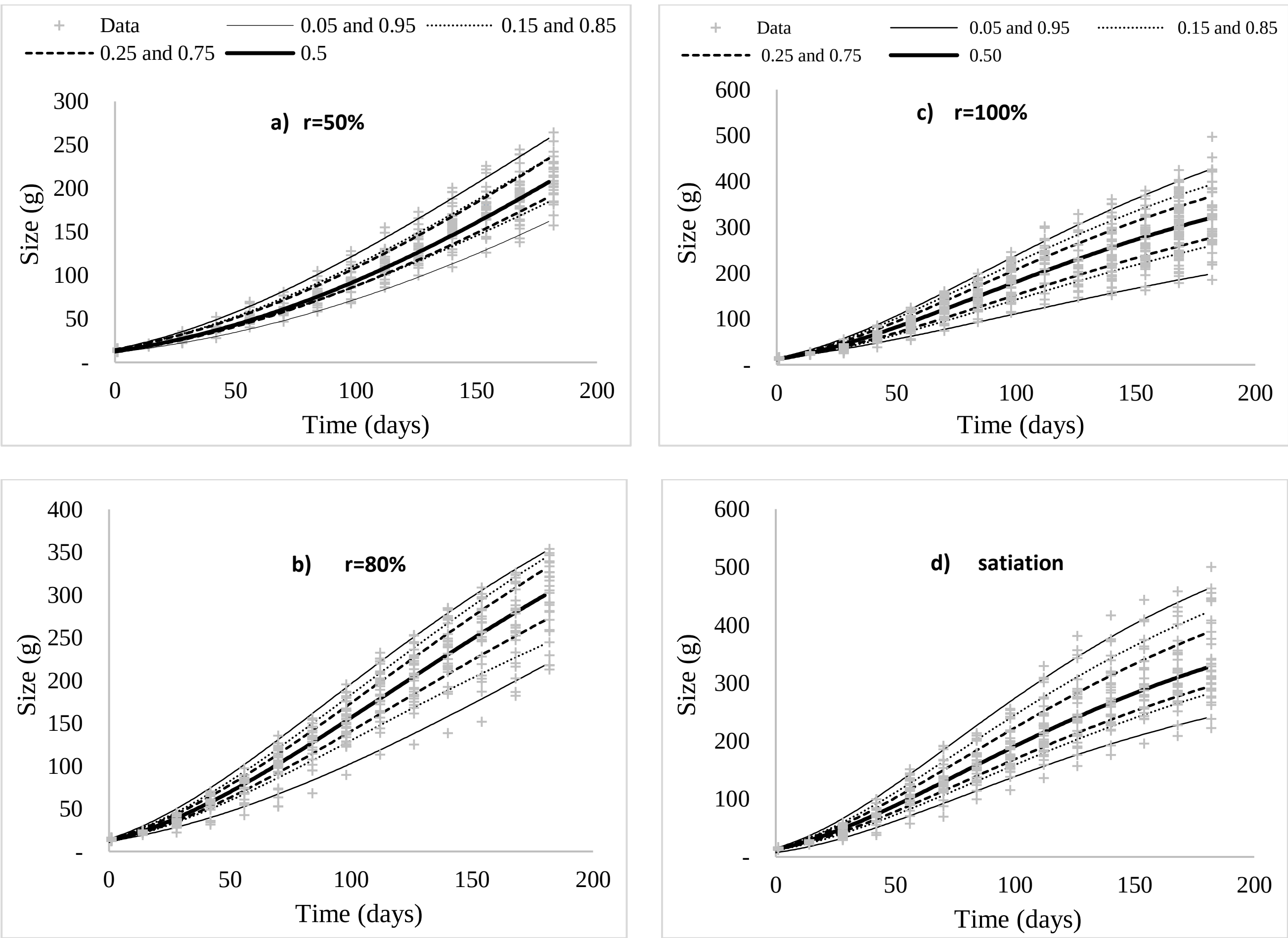

Figure 1. Quantile regression of the von Bertalanffy model from data of tilapia cultured and fed at ration size of 50% (a), 80% (b), 100% (c) and Satiation (d).

comply with the method of least squares since it only assigns a mean model to represent the growth of the organisms, but this means a loss in information, particularly of the individual growth variation (Grosjean *et al*., 2003).

Quantile Regression and Size Heterogeneity

Similar to this study, Gasca-Leyva *et al*. (2008) and Borrego-Kim *et al*. (2020a,b) in tilapia (*Oreochromis niloticus*) and Briceño et al. (2010) in *Octopus maya* also didn't incorporate the effect of a technical-biological variable with an influence over dispensing of the initial sizes, where growth heterogeneity is later on attributed (Peacor *et al*., 2007). However, they indicate that size dispersion is an occurring phenomenon even in homogenous cultures (Borrego-Kim *et al*., 2020a,b; Mayer *et al*., 2009). Results from the quantile curves observed in this study show an increase in the growth variation of tilapia when increasing the ration size. This agrees with the observations of Araneda *et al*. (2018) on growth of shrimp cultured at high densities, Domínguez-May *et al*. (2011) on growth of the tilapia at ration sizes different, Estruch *et al*. (2017) and Mayer *et al*. (2009) on growth of the gilthead seabream as a function of temperature, even though factors were not included in the proposed growth equations in this study, due to a more direct and parsimonious application of QR by the producers (Cai *et al*., 2018). However, it is evident that the quantile regression based on the selected model (vB), shows behavior that allows classifying the batches according to the growth patterns of each quantile.

This work differs from the previously mentioned studies because it assumes that the distribution of the organisms is independent of their size (Borrego-Kim *et al*., 2020a). This could be in conflict with the Peacor theory which has been approached in other studies of tilapia during the growth stabilization period (Domínguez *et a*l., 2011; Gasca-Leyva *et al*., 2008). These percentage shifts were present in all rationings except Satiation, thus indicating that a reduction the rationing size can lead to higher competition for food between the individuals and consequently generate size heterogeneity. A similar pattern was observed by Ribeiro *et al*. (2015), where fish were fed three times per day and displayed low size heterogeneity. This information corroborates that a high feeding frequency allows food access to most fish and consequently reaches a homogeneous growth in comparison to individuals fed only one time per day.

QR can simultaneously determine several growth parameters due to its versatility, and this demonstrates

Table 3. Validation of the von Bertalanffy simulation model for tilapia

Quantile (τ)	Statistical indicators						
	RECM	IF	Theil (U)	UM	UV	UC	AIC
r=50%							
0.05	3.31	0.9953	0.0317	0.1430	0.0009	0.8562	19.64
0.15	3.58	0.9957	0.0194	0.6484	0.1020	0.2568	20.60
0.25	4.44	0.9944	0.0285	0.4390	0.1874	0.3869	23.23
0.5	4.80	0.9944	0.0293	0.4013	0.0967	0.5090	24.18
0.75	5.80	0.9929	0.0289	0.5140	0.1373	0.3585	26.46
0.85	4.64	0.9959	0.0322	0.0054	0.0479	0.9501	23.75
0.95	5.00	0.9961	0.0247	0.3817	0.1227	0.5044	24.67
R=80%							
0.05	9.16	0.9808	0.0638	0.1439	0.0053	0.8512	32.02
0.15	8.24	0.9884	0.0501	0.1419	0.0115	0.8474	30.74
0.25	4.73	0.9968	0.0269	0.0828	0.0867	0.8358	23.98
0.5	6.01	0.9959	0.0313	0.0329	0.0001	0.9670	26.90
0.75	6.43	0.9963	0.0304	0.0144	0.0729	0.9180	27.71
0.85	6.04	0.9970	0.0277	0.0068	0.0468	0.9498	26.96
0.95	4.27	99.85	0.0188	0.0438	0.0691	0.8920	22.74
r=100%							
0.05	10.41	0.9762	0.0700	0.2705	0.6217	0.1522	33.58
0.15	8.00	0.9902	0.0495	0.0016	0.0016	0.9969	30.37
0.25	10.54	0.9852	0.0604	0.0140	0.0003	0.9857	33.73
0.5	13.22	0.9826	0.0646	0.0302	0.0113	0.9594	36.49
0.75	10.64	0.9917	0.0447	0.0400	0.0028	0.9574	33.84
0.85	9.94	0.9938	0.0394	0.0092	0.0372	0.9562	33.02
0.95	11.42	0.9934	0.0395	0.0842	0.2323	0.7001	34.71
Satiation							
0.05	7.57	0.9902	0.0385	0.3889	0.0141	0.5980	29.71
0.15	4.09	0.9978	0.0225	0.0524	0.0844	0.8692	22.22
0.25	3.74	0.9984	0.0195	0.0646	0.0071	0.9288	21.12
0.5	4.37	0.9982	0.0209	0.0067	0.0310	0.9645	23.03
0.75	6.47	0.9972	0.0262	0.0008	0.0000	0.9992	27.79
0.85	6.91	0.9975	0.0246	0.0529	0.1793	0.7807	28.60
0.95	10.09	0.9957	0.0323	0.0674	0.2418	0.7081	33.20

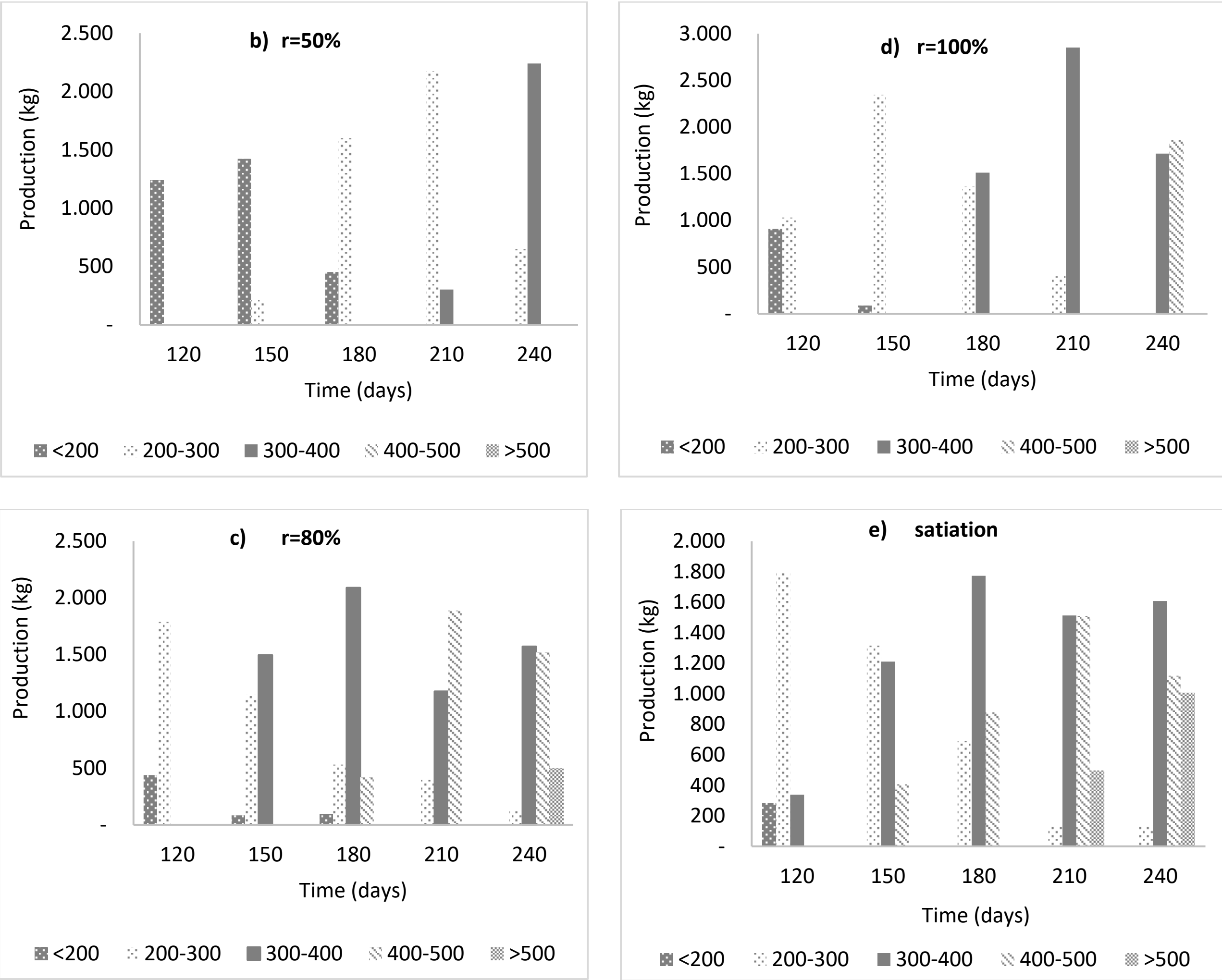

Figure 2. Biomass distribution of tilapia according to harvest days and sizes classified by quantiles 0.5,0.15,0.25,0.50,0.85, 0.95 for the 50 – Satiation ration size.

an advantage over the least squares method (Grosjean *et al*., 2003). QR shows a correlation between the parameters of the tilapia growth model (Grosjean *et al*., 2003), mainly from the asymptotic weight (W_∞) and the growth rate (k). Overall, we observed that the higher the asymptotic weight, the lower the growth rate. One possible explanation for this is that higher W_∞ values could compensate for lower growth rates. That is, there could be a higher rate of growth inhibition in the presence of a higher lack of feed, as in the case of 50% ration size, and in the opposite case higher ration sizes as Satiation. W_∞ were lower than the quantiles below the median and vice versa to quantiles above the median at rationings of 100% and Satiation. Whereas W_∞ at rationings of 50% and 80% had a reversed relationship with higher quantiles below the median vs. quantiles above the median. This data can reflect an advantage of the QR over the least square method since the growth rate in the latter is the same throughout all individuals, but the QR method can determine a growth rate for each size group and consequently show the possible effects on the inhibition of large individuals over the ones with smaller size regulated by size rationing (Grosjean *et al*., 2003).

Simulation to Commercial Size

This study has proposed the validation of growth models through the Theil index in quantile curves with the aim of using these models to simulate data in a larger time frame (Pyndick & Rubinstein, 1998; Power, 1993). The method of this study differs from validations performed by Estruch *et al*. (2017), although the statistical value employed by these authors is also found between 0 and 1 as the Theil index and its interpretation are similar. Quantile curves of tilapia in this study fulfilled the Theil requirement of less than 0.20 (Table 3). Once validated, both growth models were used to simulate different harvest strategies and different market size for production estimates (Figure 2). This was also carried out by Estruch *et al*. (2017) with gilt-head bream, although these authors studied the economic production of the quantiles. They conclude that generated data from quantiles is more robust than average models and that it can be used in decision-making for aquaculture management; for instance, classification by harvest size to have a better estimate of the population biomass (distribution) within a given time (Estruch *et al*., 2017; Mayer *et al*., 2009; Vicenzi *et* *al*., 2014), as it was demonstrated in this study.

Another advantage of this analysis can be exploited by species that have prizes defined by the market. With this method, producers could strive toward better financial conditions. Additionally, Mayer *et al*. (2009) found few changes in the biomass of homogenous lots, but heterogenous lots were characterized by important changes in fish distribution through time. A significant increase in size variability and an increase in the distance between quantiles through time is observed in this study in Figure 2a,b.

Therefore, this type of analysis (QR) can provide a positive impact on the management of tilapia production, since by knowing the size distribution of individuals, farmers can plan management strategies, such as partial harvests to reduce the effect of density on the growth of organisms, for the purpose of splitting or obtaining capital. At the same time, this may result in a marginal decrease in costs. However, to be certain of this, future experimentation may be required. This opens up new research possibilities to strengthen aquaculture, not only for tilapia, but also for other farmed species. The application of this tool could also allow us to strengthen productive monitoring and control systems to evaluate Key Performance Indicators (KPIs).

Conclusions

The quantile regression analysis has demonstrated great utility in the description of size heterogeneity and, consequently, in organisms distribution by sizes and harvest time. The simple growth curves of von Bertalanffy successfully modelled the quantiles of the observed data. The ration size showed to contribute to the size heterogeneity of tilapia. This directly affects the reached biomass and its distribution according to the different market sizes, if does not account, which could have a direct effect on the economic functions such as sales revenue, production costs and economic benefits per production unit. The predictive application in this study could be used as a tool based on the business analytics and applied to real production conditions as a tool for production control and surveillance, validation for production plans and comparison between Key Performance Indicators (KPI´s) vs. observed results. According to the latter and the results, it is possible to design by means of assumptions the prescriptive analysis that would allow an aquaculture company to estimate optimal values of management variables, such as ration or seeding density, that would allow maximizing the economic benefit or minimizing the production costs of the company in particular. According to the latter and its results, it is possible to design by means of assumptions the prescriptive analysis that would allow an aquaculture company to estimate optimal values of management variables, such as ration or seeding density, that would allow maximizing the economic benefit or minimizing the production costs of the company in particular The contribution of this study could be considered as a base line for future research that takes into account economic and management factors such as harvest analysis and decision-making, partial harvest, and optimal harvest and rotation times. Lastly, the growth model here analysed through quantile regression, can easily be applied to other species cultured at similar conditions, as well as incorporate the effects of other variables with an influence over size dispersion.

Ethical Statement

Not applicable

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Author Contribution

1. Contributed substantially to the conception and design of the study, the acquisition of data, or the analysis and interpretation: Roger Domínguez-May, Eucario Gasca-Leyva.
2. Drafted or provided critical revision of the article: Roger Domínguez-May, Eucario Gasca Leyva, Gaspar Poot-López, Marcelo Araneda.
3. Provided final approval of the version to publish: Roger Domínguez-May, Gaspar Poot-López, Marcelo Araneda.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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